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Controlling Sudden Birth and Sudden Death of Entanglement at Finite Temperature

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Abstract The decoherence and the decay of quantum entanglement due to both population relaxation and thermal effects are investigated for the two qubits initially prepared in the extended Werner-like state by solving the Lindblad form of the master equation, where each qubit is interacting with an independent reservoir at finite temperature T. Entanglement sudden death (ESD) and entanglement sudden birth (ESB) are observed during the evolution process. We analyze in detail the effects of the mixedness, the degree of entanglement of the initial states and finite temperature on the time of entanglement sudden death and entanglement sudden birth. We also obtain an analytic formula for the steady state concurrence that shows its dependence on both the system parameters, the decoherence rate and finite temperature. These results arising from the combination of extended Werner-like initial state and independent thermal reservoirs suggest an approach to control the maximum possible concurrence even after the purity and finite temperature induce sudden birth, death and revival.

Keywords Entanglement sudden death (birth) · Extended Werner-like state · Stable entanglement

1 Introduction

In the past few years there has been considerable interest in the properties of entangled quantum systems. Entanglement, is now viewed as a physical resource, which provides a means to perform quantum computation and quantum communication [1–8]. Therefore, great efforts have been made to investigate entanglement characterization, entanglement control, and entanglement production in solid-state systems such as CQED and spin chains [9–16]. In particular, the Heisenberg spin chain has been used to construct a quantum computer in many

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physical systems such as quantum dots [17], nuclear spins [18], superconductor [19] and optical lattice [20, 21] based systems. By proper encoding, the Heisenberg interaction alone can support universal quantum computation [22]. However, in the real world, since each qubit is inevitably subject to decoherence and decay processes, in most cases, this effects will introduce disentanglement and eliminate the quantum coherence in the investigated local system. So it is important to consider possible degradation of any initially established entanglement, careful investigation of well-understood model systems continue to produce surprises that add to fundamental understanding. For example, Yu and Eberly [23, 24] have found that even when there is no interaction (either directly or through a correlated environment), there are certain states whose entanglement decays exponentially with time, while for other closely related states, the entanglement vanishes completely in a finite time, this striking phenomenon is the so-called entanglement sudden death (ESD). Clearly, such finite-time disappearance of entanglement can seriously affect its application in any of the above fields. Extending Yu and Eberly's work and model by considering correlated reservoirs and interactions, further investigations of different systems in cavity-QED (Jaynes-Cummings and Tavis-Cummings model) and spin chain have been made by various groups [25–36]. In particular, ESD has been observed recently in the lab for photonic qubits [37] and atomic ensembles [38]. Opposite to the currently extensively discussed ESD, entanglement sudden birth (ESB) is the creation of entanglement where the initially unentangled qubits can be entangled after a finite evolution time [39, 40]. Although the interaction with the entanglement could lead to the disentanglement of local qubits, entanglement can also be generated by coupling two qubits to a common third system. In [41], the authors have presented concrete examples of the increase of entanglement caused by the stronger interaction of a part of a composite system with its environment. A stable entangled state, in which once qubits become entangled they will never be disentangled, was also demonstrated in [42]. For a two-qubit model, they could analytically solve the master equation consisting of a Hamiltonian part, a noise channel, which is responsible for the nonvanishing entanglement in the steady state.

In those studies, influence of dissipation on entanglement dynamics of a two-qubit Heisenberg XY spin chain with special pure initial states was studied by using exact analytic solutions. It was found that the concurrence reaches a non-zero steady-state value as the time evolves. However, in two and three-spin and in many-spin, the entangled state of a spin pair is emerged only at very low temperatures $T = 20$ mK. The same extreme conditions are required to achieve a pure state, where the populations of all quantum states except only one quantum state are equaled to zero. The pure state is usually used as a starting point for the quantum computation, communication, and teleportation algorithms. To overcome the experimental problems related to very high magnetic fields and extremely low temperatures, a so-called pseudopure state was introduced. In this paper, we present an exact calculation of the entanglement dynamics between two qubits coupling with a common environment at finite temperature. The Hamiltonian for our two-qubit system has the form of the well-known Heisenberg XY model for two interacting spins in the presence of an external magnetic field, where the effective magnetic field is defined by the energy separation of the two-level system. The main purpose of the present paper is try to solve the following question: what happens to the qubits entanglement dynamics when we consider different system parameters and initial extended Werner-like state (pseudopure state) in the presence of thermal decoherence effects? We find that the appearance of entanglement sudden death or entanglement sudden birth strongly depends on the initial entangled state and the environment. In our two-qubit model system at finite temperature, we find that for any initial state, including the one in which the two qubits are initially unentangled, the system reaches a steady state of pairwise entanglement in spite of thermal decoherence effects. This result will be helpful for the generation and transportation of entanglement in real solid-state systems.

2 Heisenberg XY Model with Initial Extended Werner-Like State

We consider an anisotropic two-qubit system interacting through an XY Heisenberg coupling under the action of dissipative environments in an external magnetic field ω along the z-axis, the corresponding Hamiltonian reads

$$H = J(S_1^+ S_2^- + S_1^- S_2^+) + \Delta(S_1^+ S_2^+ + S_1^- S_2^-) + \omega(S_1^z + S_2^z) \tag{1}$$

where $J = (J_x + J_y)/2$, $\Delta = (J_x - J_y)/2$, and $S^{\pm} = S^x \pm i S^y$ are the spin raising and lowering operators, the parameter Δ describes the spatial anisotropy of the spin-spin interaction. The anisotropy parameter can be controlled by varying J_x and J_y, which may possibly be achieved for an optical lattice system, the effective external magnetic field is defined by the energy levels of our qubits. We study a system of two qubits initially entangled and interacting with uncorrelated reservoirs. However, unlike [25], in which the system is studied at temperature $T = 0$, we include the effects of an independent reservoir in the thermal equilibrium state in our system. The description of the time evolution of an open system is provided by the master equation, which can be written most generally in the Lindblad form with the assumption of weak system-reservoir coupling and Born-Markov approximation. The Lindblad equation for our case is given by [43]

$$\begin{aligned}\frac{d\rho}{dt} &= -i[H,\rho] + (\bar{n}+1)\gamma \sum_{j=1,2}\left[S_j^- \rho S_j^+ - \frac{1}{2}\{S_j^+ S_j^-, \rho\}\right] \\ &\quad + \bar{n}\gamma \sum_{j=1,2}\left[S_j^+ \rho S_j^- - \frac{1}{2}\{S_j^- S_j^+, \rho\}\right]\end{aligned} \tag{2}$$

where γ is the spontaneous decay rate of the qubits, which is supposed to be the same for the two qubits, the assumption is reasonable provided the interaction does not significantly alter the energy level separations. The first term on the right-hand side of (2) is the unitary part of the evolution, the second term corresponds to population relaxation of the qubits due to spontaneous emissions, while the last term describes the reexcitations caused by the finite temperature; and $\bar{n}$ is the mean occupation number of the reservoir. {} means anticommutator.

For the initial state of qubit-pair AB, instead of Bell-like and Werner states [44], we shall consider the following extended Werner-like state

$$\rho_{AB}(0) = p\,|\Phi\rangle_{AB\,AB}\langle\Phi| + \frac{1-p}{4} I_{AB}, \tag{3}$$

with p the purity of the initial state of qubits AB, I_{AB} the 4×4 identity matrix and

$$|\Phi\rangle_{AB} = (\sin\theta\,|11\rangle + \cos\theta\,|00\rangle)_{AB}\,, \tag{4}$$

the Bell-like state. This class of mixed state arises naturally in a wide variety of physical situations. Particularly, it includes pure Bell states as well as the well-known Werner mixed state as special cases. Obviously, the state in (4) reduces to the standard Werner mixed state when $\theta = \pi/4$ and to Bell-like pure state when $p = 1$. By dealing with the above extended Werner-like state, we are able to study the effect of mixedness of the initial entangled state. Both the Bell-like state and Werner state, and so the extended Werner-like state, belong to the so-called X-class state [45, 46] whose density matrix is of the form

$$\rho = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix} \tag{5}$$

in the two-qubit product state basis of $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$. Substituting (5) into (2), the master equation of our system, we obtain the following first-order coupled differential equations:

$$\begin{aligned}
\frac{d\rho_{11}(t)}{dt} &= -2(1+\bar{n})\gamma\rho_{11}(t) + \bar{n}\gamma(\rho_{22}(t) + \rho_{33}(t)) - i\Delta(\rho_{41}(t) - \rho_{14}(t)), \\
\frac{d\rho_{14}(t)}{dt} &= -(1+2\bar{n})\gamma\rho_{14}(t) - i(-\Delta\rho_{11}(t) + 2\omega\rho_{14}(t) + \Delta\rho_{44}(t)), \\
\frac{d\rho_{22}(t)}{dt} &= (1+\bar{n})\gamma(\rho_{11}(t) - \rho_{22}(t)) + \bar{n}\gamma(\rho_{44}(t) - \rho_{22}(t)) - iJ(\rho_{32}(t) - \rho_{23}(t)), \\
\frac{d\rho_{23}(t)}{dt} &= -iJ[\rho_{33}(t) - \rho_{22}(t)] - (1+2\bar{n})\gamma\rho_{23}(t), \\
\frac{d\rho_{33}(t)}{dt} &= (1+\bar{n})\gamma(\rho_{11}(t) - \rho_{33}(t)) + \bar{n}\gamma(\rho_{44}(t) - \rho_{33}(t)) - iJ(\rho_{23}(t) - \rho_{32}(t)0, \\
\frac{d\rho_{44}(t)}{dt} &= (1+\bar{n})\gamma(\rho_{22}(t) + \rho_{33}(t)) - 2\bar{n}\gamma\rho_{44}(t) - i\Delta(\rho_{14}(t) - \rho_{41}(t)).
\end{aligned} \tag{6}$$

Obviously, the solution of (6) depends on the initial state of the qubits, so we can solve analytically (6) for some typical initial states, including a pure, separable/entangled initial state; the pure maximally entangled state and the mixed Werner state. Since the explicit expressions of solutions of (6) are very complicated, here we skip the details and give our results in terms of figures.

In the present work, we adopt the concept of concurrence [47] as a measure of the pairwise entanglement. The concurrence $C = 0$ corresponds to a separable state and $C = 1$ to a maximally entangled state. Nonzero concurrence means that the two qubits are entangled. For a system described by the above density matrix, which can denote either a pure or a mixed state, the concurrence is

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) \tag{7}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are the eigenvalues in a decreasing order of the spin-flipped density operator R defined by $R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ with $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, $\tilde{\rho}$ denotes the complex conjugate of ρ, σ_y is the usual Pauli matrix. Then the concurrence for a state of this form is

$$C = 2\max[0, (\sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}), (\sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}})]. \tag{8}$$

3 The Detailed Analytical and Numerical Analysis and Results

In the following, we use this formalism to investigate the dynamics of entanglement under different system parameters, such as the anisotropic parameter, external magnetic field, the mixedness, the degree of entanglement of the initial states and finite temperature. The two-qubit entanglement dynamics has been previously analyzed taking pure Bell-like states as initial states at zero temperature [48, 49]. However, as said before, it is of interest to obtain the entanglement dynamics of two independent qubits, each locally interacting with a finite temperature reservoir, in the case of more general initial conditions and in particular for Werner states.

First let us consider the simple case of the standard vacuum reservoir, i.e., $\bar{n} = 0$. In this process the system interacts with a thermal bath at zero temperature. In Fig. 1, the time evolution of the concurrence as a function of t and the purity p of the initial state for various values of the parameter θ is shown in the absence of the decoherence. We observe that for the interval $0 \leq p < 1/3$, the initial entanglement is always zero in a infinite time. From Figs. 1a and 1d, if the parameter $\theta = 0, \frac{\pi}{2}$, i.e. the degree of entanglement of the initial pure states is zero, we can see that unentangled initial state generates maximally entangled states; Meanwhile, it is obvious that the concurrence C between two qubits is zero at earlier time, corresponding to the two qubits are in disentangled states. After a finite

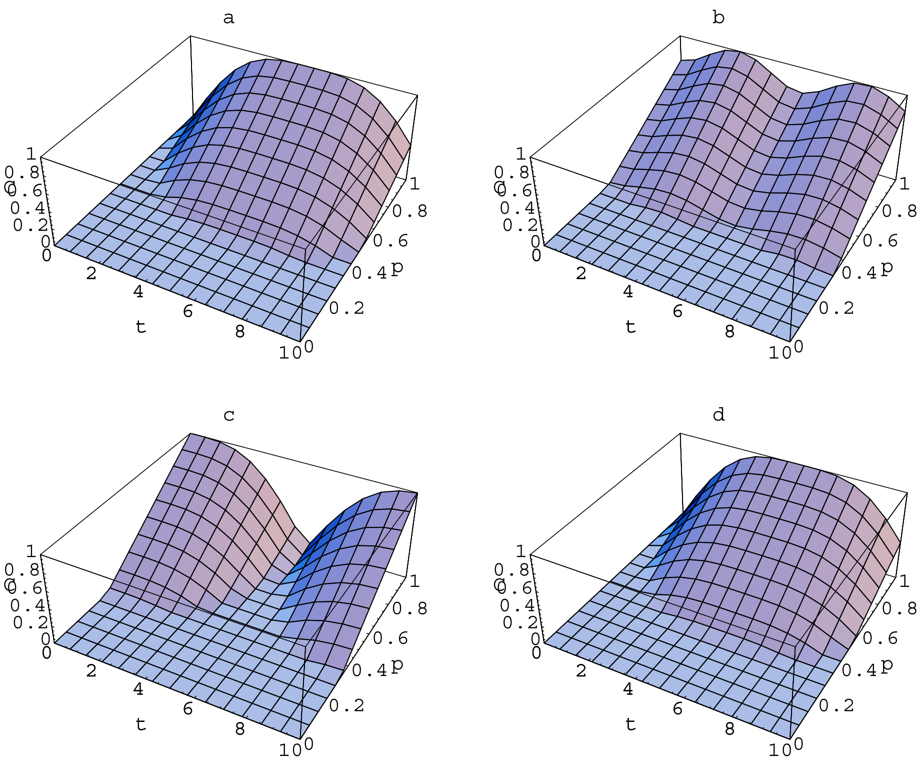

Fig. 1 The concurrence as a function of the dimensionless quantities t and p fixing the value of the external magnetic field parameter $\omega = 0.2$ with $\Delta = 0.2$, $\gamma = 0$ when the qubits are initially in different initial state. (**a**) $\theta = 0$, (**b**) $\theta = \frac{\pi}{8}$, (**c**) $\theta = \frac{\pi}{4}$, (**d**) $\theta = \frac{\pi}{2}$

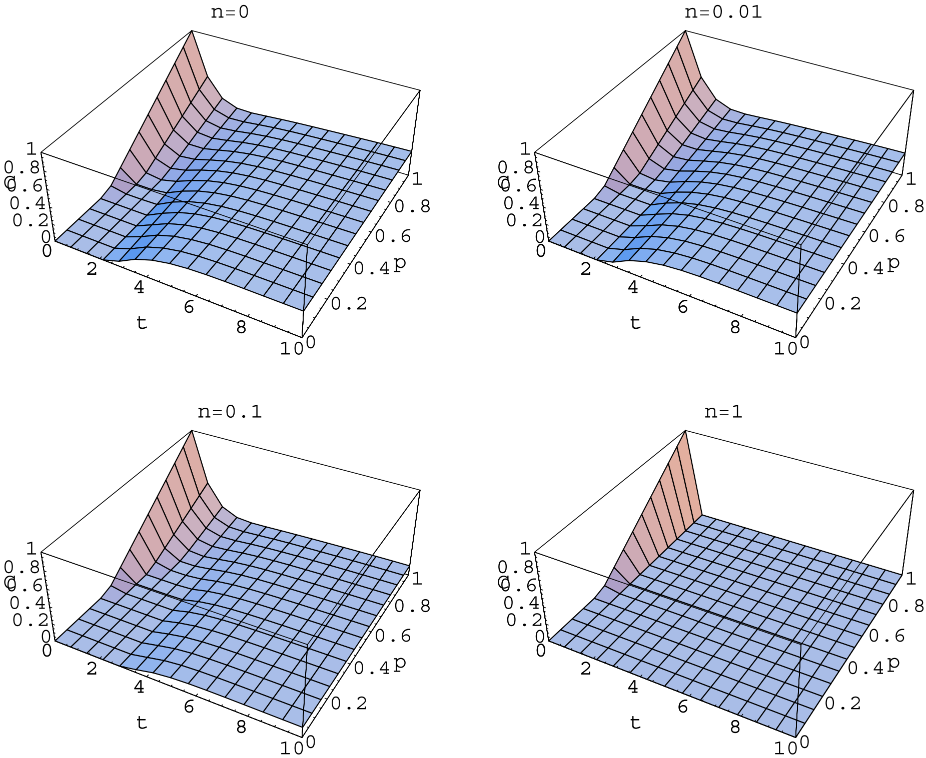

Fig. 2 The concurrence as a function of the dimensionless quantities t and p with $\Delta = 0.2$, $\omega = 0.2$, $\theta = \frac{\pi}{4}$, $\gamma = 0$ for the number of the reservoir n

time, C abruptly increases to a peak value, and the ESB occurs. While for the maximally entangled initial state $\theta = \frac{\pi}{4}$, concurrence actually goes abruptly to zero in a finite time and remains zero thereafter. That is to say, the ESD happens and the entanglement sudden death always happens when $1/3 \leq p < 1$, which is shown in Fig. 1c. When $\theta = \frac{\pi}{8}$, corresponding to the two qubits are initially in partial entangled states, we find that the ESD disappears. The whole phenomenon of sudden death and revival disappears for this initial condition, and the system shows no disentanglement. This can be also understood from the physics since the independence of qubits will be weakened by the interaction between them. Therefore, we can conclude that the parameter θ and the purity p can efficiently control the ESD. It allows us to manipulate the degree of entanglement at certain time intervals, this may be useful for the quantum information process based on entanglement.

Another aspect of interest is how the entanglement dynamics is influenced by the values of γ, which regulates the degree of coupling strength with their environment. The above analysis can easily be extended to study entanglement dynamics starting from different initial conditions and to take into account population relaxation and thermal effects. In Figs. 2, 3 and 4, the time evolution of the concurrence C for different values of parameter p and time t is plotted when the two qubits are initially in the different entangled state for $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$ with zero and finite temperature. For the zero or thermal reservoirs, we can observe that the entanglement dynamics changes in a different way, the two-qubit state can evolve into a stationary entangled state under the collective decay from initial unentangled

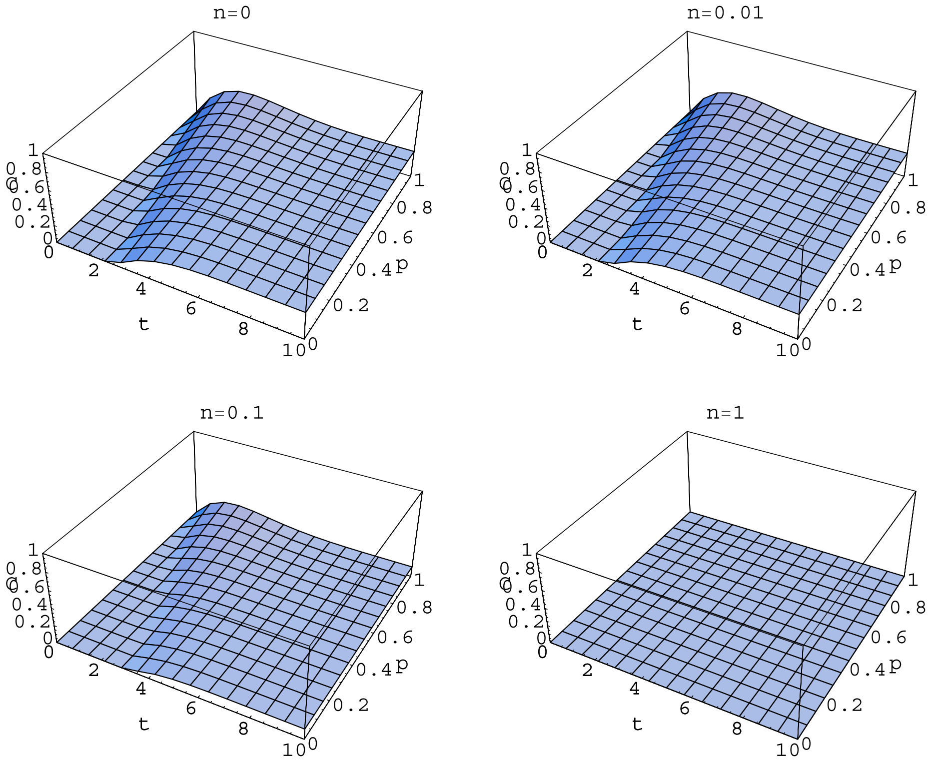

Fig. 3 The concurrence as a function of the dimensionless quantities t and p with $\Delta = 0.2$, $\omega = 0.2$, $\theta = 0$, $\gamma = 0$ for the number of the reservoir n

state. In other words, decoherence drives the qubits into a stationary entangled state instead of completely destroying the entanglement. Moreover, the stationary entanglement of two qubits is independent from different initial states of the qubits (pure or mixed state) and the degree of the purity p, while is dependence on both the system parameters, the decoherence rate and finite temperature. Figure 3 shows the dynamic behavior of the entanglement in terms of the concurrence versus t and the purity p for the initial states $\theta = \frac{\pi}{4}$, we found that when $p < 1/3$, the ESB occurs, while $1/3 \le p < 1$, we see that the effect of the zero and weak environment reservoir is to increase the disentanglement, and a stationary entangled state instead of completely disentanglement occurs. With increasing of the temperature, the concurrence changes from exponential decay to sudden death, the concurrence stays null and the revival of the entanglement disappears. In Fig. 2d, as we show, concurrence actually goes abruptly to zero in a finite time and remains zero thereafter. That is to say, the entanglement sudden death always happens in the strong qubit-environment interaction. In Fig. 2a, we have shown that the entanglement can last for an infinite period in the vacuum environment for some initial entanglement states. However, in Fig. 2d, the sudden death of entanglement always happens no matter which entanglement state the qubits are initially in. No matter what the values of other parameters are, there is no entanglement in a infinite time when the two qubits are initially in the finite temperature thermal reservoirs. The concurrences given in Fig. 3, which also shows that ESB occurs, give an evident difference in the entanglement dynamics of the two initial mixed states in Fig. 4. For mixed initial states $p < 1$, the sudden

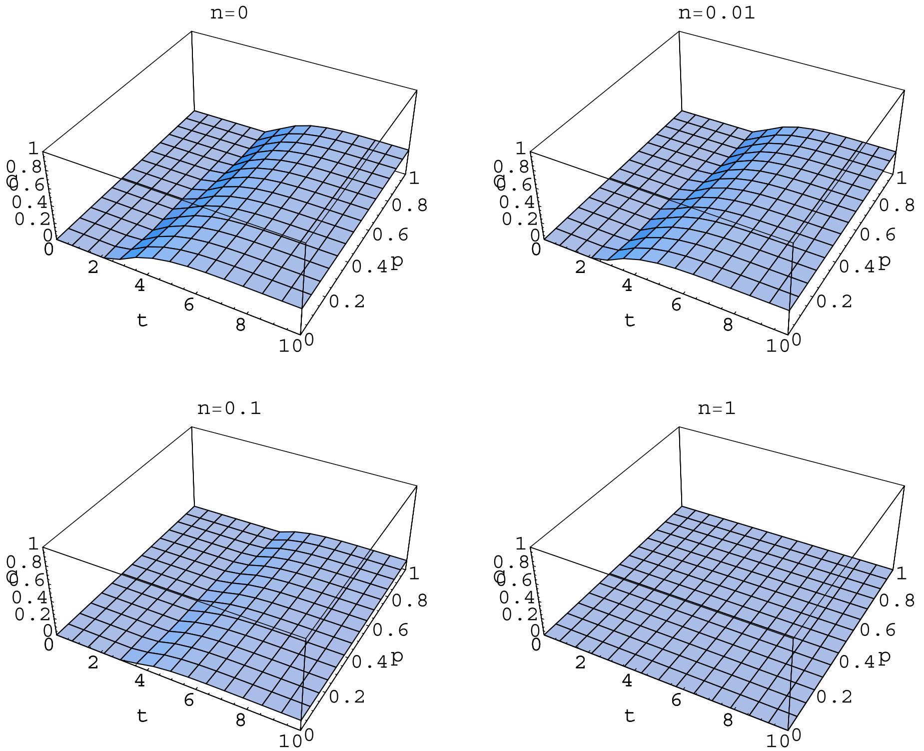

Fig. 4 The concurrence as a function of the dimensionless quantities t and p with $\Delta = 0.2$, $\omega = 0.2$, $\theta = \frac{\pi}{2}$, $\gamma = 0$ for the number of the reservoir n

birth time is prolonged with increasing the purity of the initial states ($\theta = \pi/2$), which is different from the result for $\theta = 0$, where increasing the purity can cause the length of the time intervals for nonzero entanglement to be shorter. With increasing of the mixed double excitation state component, this phenomenon of ESB is more evident. Therefore, we can conclude that the portion of the excited state component in the initial entangled state and the degree of the purity p are responsible for the ESB. As the reservoir is assumed to be finite-temperature, we can observe the concurrence changes of a slightly different character. This phenomenon is more evident as the value of n is increased. In the disentanglement dynamics of the strong dissipation environment, the lifetime, corresponding to completed death of concurrence, becomes less, and the exponential disentanglement disappears and ESD or ESB appears. Furthermore, we find the death time and birth time are prolonged and the maximal value that the entanglement can reach decreases with increasing temperature. For unentangled initial states, there appears complete disentanglement before a relatively large entanglement revival, which shows an initial decrease for both the initial mixed states but at different times, indicating a threshold value of parameter t, only above which concurrence begins to be nonzero, i.e. the quantum correlation starts to appear, the differences depends on the values of purity p. Moreover, it may cause a stable entanglement after entanglement sudden birth. The entanglement has another unusual relaxation property: different entanglement states, corresponding to different values of θ, with the same initial degrees of entanglement may evolve by different ways, some showing entanglement sudden death and

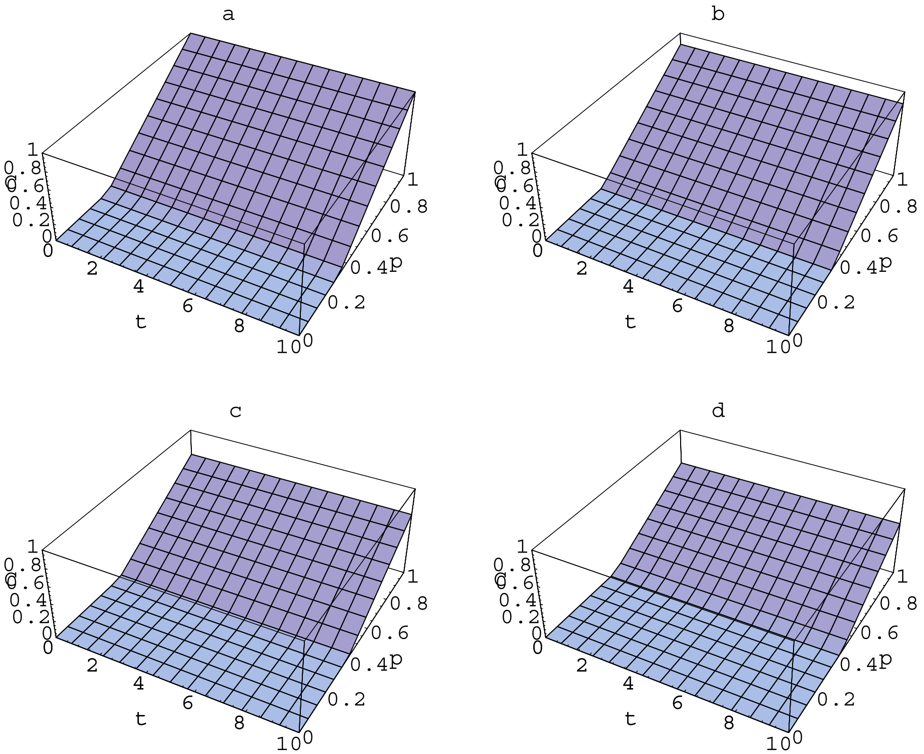

Fig. 5 The concurrence (decoherence free subspace) as a function of the dimensionless quantities t and p (**a**) $\theta = \frac{\pi}{4}$, $\Delta = 0.2$, $\omega = 0$, $\gamma = 0$, (**b**) $\theta = \frac{\pi}{3}$, $\Delta = 1$, $\omega = \frac{\sqrt{3}}{3}$, $\gamma = 0$, (**c**) $\theta = \arctan(\sqrt{2} + 1)$, $\Delta = 0.2$, $\omega = 0.2$, $\gamma = 0$, (**d**) $\theta = \arctan(3)$, $\Delta = 0.6$, $\omega = 0.8$, $\gamma = 0$

sudden birth, some not, some decaying faster, some slower. That is to say, we can prepare certain initial entanglement states to prolong entanglement time.

In particular, we also know that for states that belong initially to the decoherence free subspace (DFS) plane, the phenomenon of entanglement sudden death never occurs. Of course, if the initial state is in the DFS, the entanglement does not decay at all. In Fig. 5, we found the decoherence free subspace for this model by varying different system parameters, such as the anisotropic parameter, external magnetic field for a fixed value of the parameter θ with $\gamma = 0$. If we adjust the above parameters which satisfy $\tan\theta = \frac{\Delta}{\sqrt{\omega^2+\Delta^2-\omega}}$, we can prepare certain initial entanglement states in the decoherence free subspace (DFS) plane, the extent of decoherence free subspace (DFS) entanglement is sensitive to the spatial anisotropy, the energy level separation of the two qubits and the purity of the initial states p. Increasing the purity of the initial states can increase the extent of decoherence free subspace (DFS) entanglement. To the extent that these parameters can be varied in some particular physical realization of our model system, thus, the magnitude of decoherence free subspace (DFS) entanglement may be controlled. This result will be helpful for the generation and transportation of decoherence free subspace (DFS) entanglement in real solid-state systems.

Despite the presence of finite temperature decoherence, the results in Fig. 6 show that the concurrence reaches the same steady value at $n = 0$, after some oscillatory behavior, for a given set of system parameters regardless of the initial state of the system. It is of

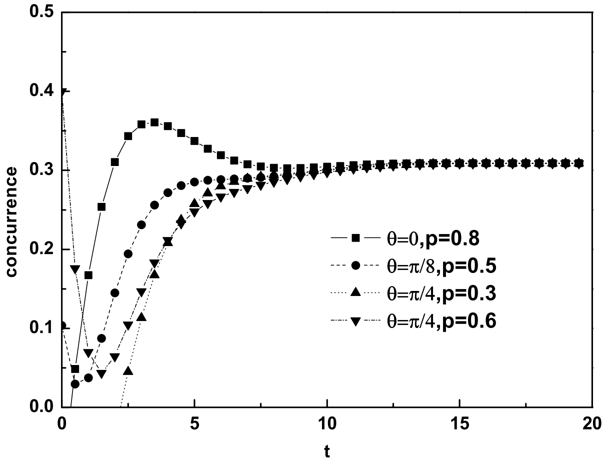

Fig. 6 The time evolution of the concurrence and steady-state concurrence for various parameter values θ and p. The parameter values for the plot are $\omega = 0.2$, $\Delta = 0.2$, $\gamma = 0.5$, $n = 0$

interest to examine how the steady state entanglement obtained for zero temperature in the prior section changes when the temperature is finite. The concurrence may be calculated for the finite temperature, by obtaining steady-state density matrix, the corresponding steady concurrence is found to be

$$C_s = \frac{2\Delta\sqrt{4\bar{\omega}^2 + (1+2\bar{n})^2} - 2\Delta^2}{(1+2\bar{n})(4(\bar{\omega}^2 + \bar{\Delta}^2) + (1+2\bar{n})^2)} - \frac{1}{2} + \frac{4\bar{\omega}^2 + (1+2\bar{n})^2}{2(1+2\bar{n})^2(4(\bar{\omega}^2 + \bar{\Delta}^2) + (1+2\bar{n})^2)}. \quad (9)$$

The steady-state concurrence is seen to depend on the system parameters ω, Δ, γ and n, while independent of the J and the initial entangled state of the qubits.

4 Discussion

In summary, we present the decoherence and the decay of quantum entanglement due to both population relaxation and thermal effects or the two qubits initially prepared in the extended Werner-like state by solving the Lindblad form of the master equation, where each qubit is interacting with an independent reservoir at finite temperature T. we find that ESD and ESB appear simultaneously, depends on the initial state, the anisotropic parameter, external magnetic field, the coupling strength with the environment, the mixedness, the degree of entanglement of the initial states and finite temperature. We also obtain an analytic formula for the steady state concurrence that shows its dependence on both the system parameters, the decoherence rate and finite temperature while independent of different initial states of the qubits (pure or mixed state). These results arising from the combination of extended Werner-like initial state and independent thermal reservoir suggest an approach to control and specify optimal values for these parameters to achieve the maximum possible concurrence even after the purity and finite temperature induce sudden birth, death and revival. We believe that our results contribute in shedding light on the delay and avoidance of ESD of initially prepared two-qubit X-states in this open quantum system and the behavior of quantum entanglement in realistic conditions, that is when the effect of the environment on the quantum system is taken into account. In physical contexts, the observation of the effects we have discussed should be achievable with the current experimental technologies.

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